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Multiple-trapping transient currents in thin insulating layers with spatially non-homogeneous trap distribution

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Abstract. The equations for multiple-trapping transport under conditions of pulse injection have been solved for a macroscopically non-homogeneous spatial trap distribution (STD). Since the influence of the STD on transient currents is the main point of interest, we deal only with two model energetic trap distributions—monoenergetic for non-dispersive transport, and exponential for dispersive transport. The expressions for transient currents obtained are discussed with respect of their utility in determining STD on the basis of experimental transients. Analytically calculated transient currents agree perfectly with the results obtained from the Monte Carlo simulation of these transients.

1. Introduction

The time-dependent equations for multiple-trapping transport under conditions of pulse injection into thin insulating layers have been successfully discussed in many papers (Zanio *et al* 1968, Teft 1967, Schmidlin 1977, Noolandi 1977, Scher and Montroll 1975, Arkchipov and Rudenko 1982, Rudenko and Arkchipov 1982a, b, Tomaszewicz and Jachym 1984). In all the previous research in the field only a homogeneous spatial trap distribution (STD) has been assumed. Such an assumption essentially simplifies the resulting formulae for transient currents but corresponds, however, to a very idealised situation. Spatial non-homogeneity of the trap distribution in a layer may arise from diffusion of atoms from contacts or ambient atmosphere, or from chemical reactions. In the case of very thin layers the region of trap non-homogeneity may be comparable with the layer thickness, so that the near-contact non-homogeneity may not be included into contact properties and should result in distinct discrepancies of the measured current–time characteristics from theoretical results obtained for a homogeneous STD.

In the present paper we solve multiple-trapping transport equations for small-signal pulse injection in insulating layers with a spatially non-homogeneous trap distribution. Depending on the energetic trap distribution, either dispersive or non-dispersive transport occurs. Because we are interested in the influence of the STD on transient currents, we deal only with two model energetic trap distributions: monoenergetic, to illustrate the influence of STD non-homogeneity on non-dispersive transients (§ 2); and exponential, to illustrate the influence of such non-homogeneity on dispersive transients (§ 3). In both cases we propose methods of determining the STD from experimental current–time characteristics. Section 4 contains concluding remarks.

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2. Non-dispersive transport

In the case of small-signal monopolar injection into a thin insulating layer, the continuity equations for concentrations of the free charge $n(x, t)$ and the trapped charge $n_t(x, t)$, assuming multiple-trapping band transport, may be written as follows:

$$\partial n(x,t)/\partial t = -\mu E(\partial n(x,t)/\partial x) - \partial n_t(x,t)/\partial t \quad (1)$$

$$\partial n_t(x,t)/\partial t = n(x,t)/\tau(x) - n_t(x,t)/\tau_d \quad (2)$$

with initial conditions

$$n(x,0) = n_0\delta(x) \quad (3)$$

$$n_t(x,0) = 0 \quad (4)$$

where μ is the microscopic mobility; E is the electric field ($E = \text{const}$); $\tau(x) = [N_0S(x)\sigma\mu E]^{-1}$ is the x-dependent average trapping time; $S(x)$ is the shape function of the trap distribution; $N_0S(x)$ is the trap concentration in x; $\sigma = \sigma_0 v_{th}/\mu E$ is the effective trapping cross section; σ_0 is the trapping cross section; v_{th} is the thermal velocity; τ_d is the average detrapping time; n_0 is the surface density of initially generated carriers; $\delta(x)$ is the Dirac function; $t \geqslant 0$; and $0 \leqslant x \leqslant L$, where L is the layer thickness. In equation (1) the diffusion term has been neglected, and in equation (2) a low trap occupation has been assumed. Equations (1)–(4) may be solved with the aid of Laplace transform technique (Appendix 1). The expression for $n(x, t)$ obtained reads

$$n(x,t) = \frac{n_0}{\mu E}\left[\delta\left(t - \frac{x}{\mu E}\right) + \Theta\left(t - \frac{x}{\mu E}\right) \times \frac{\xi I_1(\xi)}{2(t - x/\mu E)}\exp\left(-\frac{t - x/\mu E}{\tau_d}\right)\right]\exp\left(-\frac{\bar{S}(x)}{\mu E\tau_0}\right) \quad (5)$$

where

$$\xi = 2[\bar{S}(x)(t - x/\mu E)/\mu E\tau_0\tau_d]^{1/2} \quad (6)$$

$$\tau_0 = (N_0\sigma\mu E)^{-1} \quad (7)$$

and

$$\bar{S}(x) = \int_0^x S(\zeta)\,d\zeta. \quad (8)$$

Here Θ is the unit step function, and I_1 is the hyperbolic Bessel function of the first order. Equation (5) generalises the corresponding result of Zanio *et al* (1968) to the case of macroscopically non-homogeneous STD. Expression (5) integrated over the layer thickness gives the time-dependent current $j(t)$ induced in the external circuit:

$$j(t) = q\mu E\int_0^L n(x,t)\,dx/L. \quad (9)$$

Equations (5)–(9) allow one to calculate numerically transient currents for any given $S(x)$. As far as determination of the STD in a given layer from experimental transients is concerned, one could at least assume *a priori* a certain functional shape of $S(x)$ in the form of a one- or two-parameter family of functions and determine the concrete values of these by fitting experimental data to theoretical currents. However, in two limiting cases of shallow and deep traps very simple expressions for transient currents may be obtained, enabling us to determine the STD immediately from experimental characteristics.

In the case of shallow trapping, which corresponds to $s\tau_d \ll 1$ (Appendix 1), $n(x, t)$ is described by a δ-like packet of carriers:

$$n(x,t) = (n_0/\mu E)\delta\{t - (1/\mu E)[x + N_0\bar{S}(x)\tau_d \sigma\mu E]\} \tag{10}$$

and thus

$$j(t) = j_0\{1 + N_0 S[x^*(t)]\tau_d \sigma\mu E\}^{-1}. \tag{11}$$

In equations (10) and (11) $j_0 = n_0\mu Eq/L$, and $x^*(t)$ is the actual position of the carrier packet. So $x^*(t)$ is the solution of the equation

$$x + N_0\bar{S}(x)\tau_d\sigma\mu E = \mu Et. \tag{12}$$

From equation (11) we get an expression suitable for the determination of the STD, provided $j(t)$ has been measured:

$$N_0 S[x^*(t)] = [j_0 - j(t)]/j(t)\tau_d\sigma\mu E. \tag{13}$$

Equation (13) gives the trap concentration at a distance $x^*(t)$ from the injecting contact, that is in the actual position of drifting wall of carriers. However, $x^*(t)$ may be immediately related to the absolute time-independent coordinate x. In particular, the charge $Q(t)$ induced on a unit surface of contact up to the moment t amounts to (Ramo 1939)

$$Q(t) = n_0 q x^*(t)/L \tag{14}$$

which correlates time t to the actual position $x^*(t)$ of drifting carriers. Thus having measured $Q(t)$ the right-hand side of equation (13) equals the trap density in $x = LQ(t)/qn_0$. If σ and/or τ_d is not known, equations (13) and (14) allow one to determine only the shape function $S(x)$ normalised to 1 in $x = 0$ and the product $N_0\sigma\tau_d$. Figures 1–3 illustrate transient currents in the case of shallow trapping for several shape functions $S(x)$:

$$S(x) = \exp(-x/D) \tag{15}$$

$$S(x) = \exp[-(L - x)/D] \tag{16}$$

$$S(x) = \exp(-x/D_1) + \exp[-(L - x)/D_2] \tag{17}$$

$$S(x) = \exp[-(x - L/2)^2/D^2] \tag{18}$$

of various degrees of STD non-homogeneity. D, D_1 and D_2 in equations (15)–(18) are parameters. The full curves in figures 1–3 have been obtained by numerical integration of the expressions (5)–(9): the broken curves correspond to equation (11) together with (12), the latter being solved numerically. The points in figures 1–3 represent the results of Monte Carlo simulation of corresponding transients according to the well known algorithm, proposed originally by Silver *et al* (1970), here, however, generalised by allowing the average trapping time $\tau(x)$ to depend on x. As is easily seen, in spite of the rather crude approximation made to get (10) (Appendix 1), transients given by (11) agree perfectly with both the exact solutions of the transport equations and the Monte Carlo simulation up to the moment when the fastest carriers reach $x = L$. Thus equations

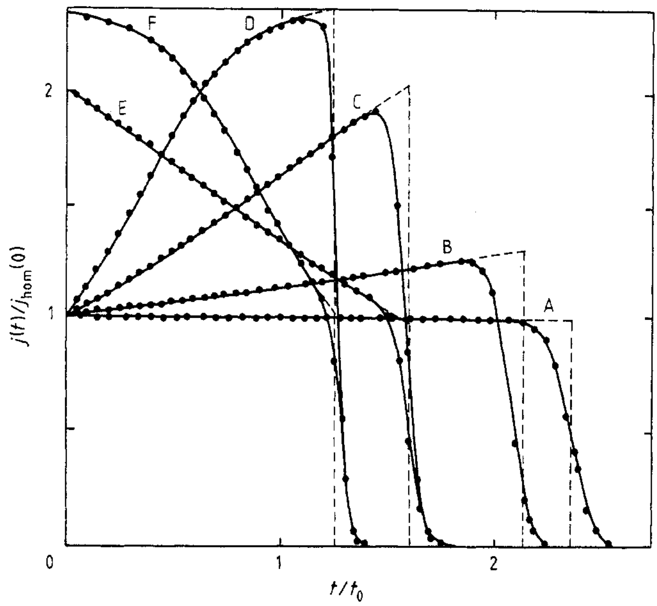

Figure 1. Shallow trapping transient currents for different degrees of non-homogeneity L/D in STDs (15) and (16): A–D, $S(x) = \exp(-x/D)$, $L/D = 0$ (A), 0.5 (B), 2.0 (C), 5.0 (D); E, F, $S(x) = \exp[-(L-x)/D]$, $L/D = 2.0$ (E), 5.0 (F). $\tau_0 = 0.002t_0$ ($t_0 = L/\mu E$), $\tau_d = 2.71\tau_0$. The meaning of the curves and points is described in the text.

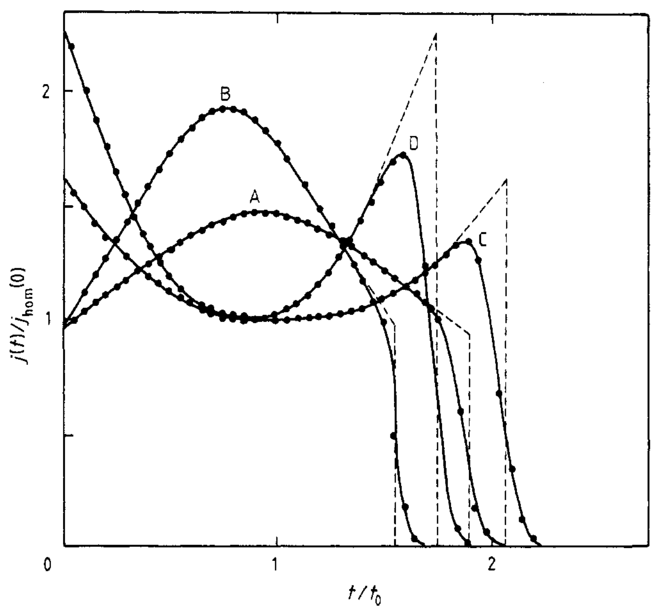

Figure 2. Shallow trapping transient currents for different degrees of non-homogeneity L/D in STDs (17) and (18): A, B, $S(x) = \exp(-x/D_1) + \exp[-(L-x)/D_2]$, $L/D_1 = L/D_2 = 3.0$ (A), 5.0 (B); C, D, $S(x) = \exp[-(x-L/2)^2/D^2]$, $L^2/D^2 = 4.0$ (C), 12.0 (D). $\tau_0 = 0.002t_0$ ($t_0 = L/\mu E$), $\tau_d = 2.71\tau_0$. The meaning of the curves and points is described in the text.

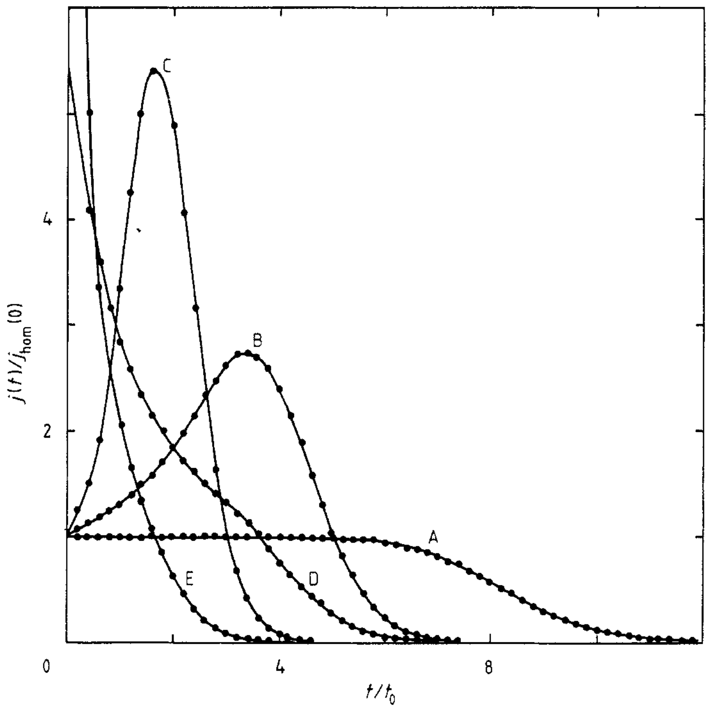

Figure 3. Transient currents for different degrees of non-homogeneity L/D in STDs (15) and (16): A–C, $S(x) = \exp(-x/D)$, $L/D = 0$ (A), 2.0 (B), 5.0 (C); D, E, $S(x) = \exp[-(L-x)/D]$, $L/D = 2.0$ (D), 5.0 (E). $\tau_0 = 0.01t_0$, $\tau_d = t_0$ ($t_0 = L/\mu E$). The meaning of the curves and points is described in the text.

(13) and (14) make it possible to determine STD from experimental current–time characteristics in the range $0 \leqslant x \leqslant L - w$, w approximately being the width of the Gaussian packet of drifting carriers.

In the case of deep trapping ($s\tau_d \gg 1$), for $t \leqslant t_0$, where t_0 is the trap-free time of flight, one gets (Appendix 1)

$$n(x, t) = (n_0/\mu E)\exp[-\bar{S}(x)/\mu E\tau_0]\delta(t - x/\mu E) \tag{19}$$

and thus

$$j(t) = j_0 \exp[-\bar{S}(\mu E t)/\mu E\tau_0]. \tag{20}$$

Equation (20) may be used for immediate determination of the STD:

$$N_0 S(\mu E t) = -[1/\sigma\mu E j(t)]\,\mathrm{d}j(t)/\mathrm{d}t \tag{21}$$

provided σ and μ are known.

Equations (13), (14) and (21) allow determination of the STD from experimental data only in two limiting cases of shallow and deep traps, respectively. No simple equation, suitable for determination of the STD in the intermediate region ($\tau_d \simeq \tau \simeq t_0$), has been found. However, traps may be made effectively shallower or deeper at higher or lower temperatures, respectively. Thus it seems that the simplified expressions (13), (14) and (21) presented above could make STD estimation possible for a relatively wide class of materials.

3. Dispersive transport

In the case of dispersive transport the continuity equations to be solved are (1) and (cf. (2))

$$\partial n_t'(x,t,\mathscr{E})/\partial t = C(\mathscr{E})N_0S(x)f(\mathscr{E})n(x,t) - n_t'(x,t,\mathscr{E})/\tau_{\mathrm{d}}(\mathscr{E}) \tag{22}$$

with the initial conditions (3) and (4), where $n_t'(x,t,\mathscr{E})$ is the density of the carriers trapped in the depth interval $(\mathscr{E}, \mathscr{E}+\mathrm{d}\mathscr{E})$, $\mathscr{E}$ being measured down from the upper edge $\mathscr{E}_0$ of the energetic trap distribution; $f(\mathscr{E})$ is the energetic trap distribution, here assumed to be

$$f(\mathscr{E}) = (1/kT_{\mathrm{c}})\exp(-\mathscr{E}/kT_{\mathrm{c}}) \tag{23}$$

where k is the Boltzmann constant, T_{c} is the characteristic temperature, $C(\mathscr{E})$ is the carrier capture coefficient, $\tau_{\mathrm{d}}(\mathscr{E}) = 1/\nu(\mathscr{E}_0)\exp(\mathscr{E}/kT)$ is the mean detrapping time from the energetic level $\mathscr{E}_0$, $\nu(\mathscr{E}_0) = \nu_0\exp(\mathscr{E}_0/kT_{\mathrm{c}})$ is the effective frequency factor, ν_0 is the frequency factor and T is temperature.

Following the considerations of Tomaszewicz and Jachym (1984), extended, however, by the change of variables $x' = N_0\bar{S}(x)/\mu E$ and $\bar{n}_t(x,t) = n_t(x,t)/N_0S(x)$, the carrier concentration in the conduction band may be expressed by the following approximate formula:

$$n(x,t) = (n_0/\mu E)\,\partial\exp[-\bar{S}(x)\Phi(t)/\mu E]/\partial t \tag{24}$$

where

$$\Phi(t) = \int_0^\infty C(\mathscr{E})N_0f(\mathscr{E})\exp[-t/\tau_{\mathrm{d}}(\mathscr{E})]\,\mathrm{d}\mathscr{E}. \tag{25}$$

Equations (24) and (25) together with (9) suffice to calculate transient currents. According to the approximations described precisely by Tomaszewicz and Jachym (1984), equation (24) is valid for highly dispersive transport. Exact solution of equations (1) and (22) may be obtained formally with the aid of the Laplace transform technique. In particular, from (1) and (22) one gets the following expression for the time transform $\tilde{n}(x,s)$ of the free carrier concentration $n(x,t)$ (Appendix 2):

$$\tilde{n}(x,s) = (n_0/\mu E)\exp\{-(s/\mu E)[x + \bar{S}(x)\Phi(s)]\}\Theta(x) \tag{26}$$

where $\Phi(s)$ is the Laplace transform of (25), for the exponential energetic trap distribution (23) given by

$$\Phi(s) = \frac{CN_0\alpha}{s}\int_0^1 \frac{u^{\alpha-1}\,\mathrm{d}u}{1+\nu u/s} \tag{27}$$

where $u = \exp(-\mathscr{E}/kT)$, $\alpha = T/T_{\mathrm{c}}$ and C has been assumed to be constant. Numerical calculation of $j(t)$ consists of evaluation of (27), inversion of (26) and inserting the result into (9). The first of these steps may be performed to some extent analytically. In particular, if $1/\alpha$ is an integer, the substitutions $w = \nu u/s$ and $z = w^{\alpha}$ in equation (27) yield

$$\Phi(s) = \frac{CN_0(s/\nu)^{\alpha}}{s}\int_0^{(s/\nu)^{-\alpha}} \frac{dz}{1 + z^{1/\alpha}} \tag{28}$$

which contains the known integral (e.g. Ryżyk and Gradsztejn 1964):

$$\int \frac{dz}{1+z^m} = \frac{1}{m}\ln(1+z) - \frac{2}{m}\sum_{k=0}^{(m-3)/2} P_k \cos\left(\frac{(2k+1)\pi}{m}\right) + \frac{2}{m}\sum_{k=0}^{(m-3)/2} Q_k \sin\left(\frac{(2k+1)\pi}{m}\right) \tag{29}$$

for odd integers, and

$$\int \frac{dz}{1+z^m} = \frac{1}{m}\sum_{k=0}^{m/2-1} P_k \cos\left(\frac{(2k+1)\pi}{m}\right) + \frac{1}{m}\sum_{k=0}^{m/2-1} Q_k \sin\left(\frac{(2k+1)\pi}{m}\right) \tag{30}$$

for even integers, where

$$P_k = 0.5\ln\left[z^2 - 2z\cos\left(\frac{(2k+1)\pi}{m}\right) + 1\right] \tag{31}$$

and

$$Q_k = \tan^{-1}\left(\frac{z - \cos[(2k+1)\pi/m]}{\sin[(2k+1)\pi/m]}\right). \tag{32}$$

On the other hand, for any real α from the interval (0, 1), in the limit of large times (small s), $\Phi(s)$ may be approximated by

$$\Phi(s) = CN_0(\gamma\nu)^{-\alpha}\frac{\Gamma(-\alpha+1)}{kT_c s^{-\alpha+1}} \tag{33}$$

where $\gamma = \exp(\mathscr{C}) = 1.786$, $\mathscr{C}$ is the Euler constant, and Γ is the Euler function. The development of (33) consists of application of (23) to equations (B5) and (B6) of Tomaszewicz and Jachym (1984) and calculating the Laplace transform $\mathscr{L}$ using the well known formula $\mathscr{L}[t^r] = \Gamma(r+1)/s^{r+1}$, $\mathrm{Re}\, r > -1$. In figure 4 we compare the analytically calculated dispersive transients with our previous Monte Carlo results (Rybicki and Chybicki 1988). As expected, the currents calculated from (9) and the approximate solution (24) and (25) agree with the simulation results (treated here as the exact transients) for small α ($\alpha < 0.33$). The numerically obtained current–time characteristics agree with the simulation results for all α.

As argued in Rybicki and Chybicki (1988), the shape of the transient multiple-trapping currents are not very sensitive to the actual shape of the STD, particularly in the case of highly dispersive transport. Practically, transient currents depend only on the STD in the region close to the injecting contact, where the drifting packet of carriers is relatively mobile. Thus, as far as the estimation of the STD shape on the basis of transient currents is concerned, some simple, approximate formulae describing initial portions of transients should be useful. With the aid of the saddle-point method, the small-time

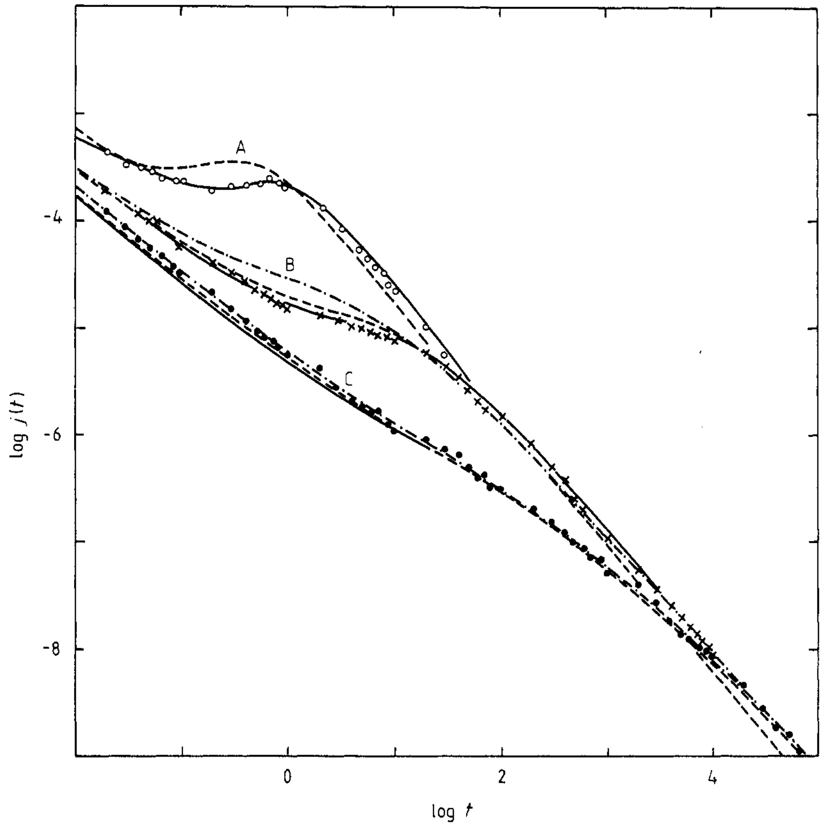

Figure 4. Theoretical dispersive transients compared to Monte Carlo simulation results. $S(x) = \exp(-x/D)$, $L/D = 5.0$, $C = 10^{-13}\,\mathrm{m^3\,s^{-1}}$ for: (A) $\alpha = 0.5$, $\nu = 5 \times 10^3\,\mathrm{s^{-1}}$; (B) $\alpha = 0.33$; (C) $\alpha = 0.2$, $\nu = 10^5\,\mathrm{s^{-1}}$. Points, Monte Carlo simulation; full curves, equations (9), (26)–(32); broken curves, equations (9), (26) and (33); chain curves, equations (9), (24) and (25).

expressions for $j(t)$ may be obtained from (9) and (24). For $S(x)$ given by equation (15) one gets (Appendix 3)

$$j(t) = -[qn_0\mu E(2\pi)^{1/2}/Le][\mathrm{d}\ln\Phi(t)/\mathrm{d}t][\Phi(t) - \mu E/D]^{-1} \tag{34}$$

$e = 2.71$. The corresponding formula for the STD (16) may be immediately written by substituting $-D$ in place of D in (34) and $N_0\exp(-L/D)$ in place of N_0 in (25). The formula for STD (17) with $D_1 = D_2 = D$ reads

$$j(t) = -\frac{qn_0(2\pi)^{1/2}}{Le}\frac{\mathrm{d}\ln\Phi(t)}{\mathrm{d}t}\left\{2\frac{\Phi(t)}{\mu E}\times\exp\left(\frac{-L}{2D}\right)\cosh\left[\sinh^{-1}\left(\frac{\mu E\exp(L/2D)}{2D\Phi(t)} - \sinh\left(\frac{L}{2D}\right)\right)\right]\right\}^{-1}. \tag{35}$$

Equation (35) for $L/D \ll 1$ and $L/D \gg 1$ reduces to the homogeneous STD expression with trap concentration $2N_0$, and to equation (34), respectively. The approximate formulae (34) and (35) for STDs (15)–(17) agree with the exact solutions in the time range from several to about 10^2 trap-free times of flight. On inspection, they account for the qualitative dependence of transients on the STD non-homogeneity, presented in Rybicki and Chybicki (1988). Equation (34) may be used for the estimation of the trap distribution parameters in the near-contact region from experimental data, the STD being approximated by (15), provided $\Phi(t)$ (dependent on C, ν and the energetic trap distribution) is known for the material considered.

4. Concluding remarks

In the present paper we have formally generalised the well known simple analytical description of multiple-trapping transient currents to the case of layers with a spatially non-homogeneous trap distribution, for both non-dispersive and dispersive transport. It has been demonstrated that the spatial non-homogeneity in the trap distribution may lead to essential deviations from transients expected for homogeneous layers. Although the solutions of the transport equations are rather complicated, and require some numerical work to calculate transient currents, some approximate formulae may be given. These are suitable for determining the spatial trap distribution over almost all the layer thickness in the non-dispersive case, and estimating it in the near-contact region in the case of dispersive transport.

Transients similar to those of figures 1–4, when obtained in experiments, may be explained within the framework of spatial non-homogeneity of a layer not only by assuming spatial variations of the trap concentration, but also in at least two other ways. In particular, it is easy to determine the spatial variations of depths of traps with x-independent concentration, which lead to transients identical to those obtained for constant-depth traps of x-dependent concentration. One can also easily find a constant in-built electrical field, which would modify the local drift velocity in such a way that the transient pulse assumes exactly the same shape as shown in the present work. Corresponding formulae, and analysis of the unambiguity of the experimental data interpretation in terms of the layer non-homogeneity, will be presented in a forthcoming paper.

Acknowledgments

We wish to express our thanks to Dr W Tomaszewicz and Dr H Samplawski for their interest in and helpful discussions on our work. Support from CPBP 0108 E3 is kindly acknowledged.

Appendix 1

We shall solve here equations (1)–(4) with the aid of the Laplace transform technique.

Time Laplace transforms of equations (1) and (2), together with initial conditions (3) and (4), are as follows:

$$\mu E(\partial \tilde{n}(x,s)/\partial x) + s\tilde{n}(x,s) + s\tilde{n}_t(x,s) = n_0\delta(x) \tag{A1.1}$$

$$\tilde{n}_t(x,s) = [1/\tau(x)](s + 1/\tau_\mathrm{d})^{-1}\tilde{n}(x,s). \tag{A1.2}$$

Substituting $\tilde{n}_t(x, s)$ from (A1.2) into (A1.1) and integrating the latter with respect to x one gets

$$\tilde{n}(x, s) = \frac{n_0}{\mu E} \exp\left[-\frac{s}{\mu E}\left(x + \frac{\bar{S}(x)}{\tau_0(s + 1/\tau_d)}\right)\right] \tag{A1.3}$$

$$\tilde{n}_t(x,s) = \frac{n_0}{\tau(x)\mu E}\left(s + \frac{1}{\tau_d}\right)^{-1} \exp\left[-\frac{s}{\mu E}\left(x + \frac{\bar{S}(x)}{\tau_0(s + 1/\tau_d)}\right)\right] \tag{A1.4}$$

where $\bar{S}(x)$ and τ_0 are given by (9) and (7), respectively.

The problem is now reduced to finding the inverse Laplace transforms of (A1.3) and (A1.4). Because we are interested in the s-dependence of (A1.3) and (A1.4), we shall introduce the following notation: $A = x/\mu E$, $B = \bar{S}(x)/\mu E\tau_0$, $\alpha = n_0/\mu E$ and $\beta = 1/\tau_d$. Now we have

$$\tilde{n}(x, s) = \alpha\, e^{-As} \exp[-Bs/(s + \beta)] \tag{A1.5}$$

$$\tilde{n}_t(x,s) = [\alpha/\tau(x)][1/(s + \beta)]\, e^{-As} \exp[-Bs/(s + \beta)]. \tag{A1.6}$$

Let us consider (A1.5): for $\mathrm{Re}\, s > -\beta$, which is fulfilled in our case, we have

$$\tilde{n}(x, s) = \alpha\, e^{-As-B} \sum_{k=0}^{\infty} \frac{B^k}{k!} \frac{\beta^k}{(s + \beta)^k} = \alpha\, e^{-As-B}\left(1 + \sum_{k=1}^{\infty} \frac{(B\beta)^k}{k!(s + \beta)^k}\right)$$

$$= \alpha\, e^{-As-B}\left(1 + \sum_{k=0}^{\infty} \frac{(B\beta)^{k+1}}{(k + 1)!(s + \beta)^{k+1}}\right)$$

$$= \alpha e^{-As-B} + \alpha\beta B e^{-As-B} \sum_{k=0}^{\infty} \frac{(B\beta)^k}{k!(k+1)!} \frac{k!}{(s+\beta)^{k+1}}.$$

Term-by-term inversion yields

$$n(x,t) = \alpha e^{-B}\delta(t - A) + \alpha\beta B e^{-B} \sum_{k=0}^{\infty} \frac{(B\beta)^k}{k!(k+1)!}(t - A)^k e^{-\beta(t-A)}\Theta(t - A)$$

$$= \alpha\, e^{-B}\delta(t - A) + \alpha\beta B\, e^{-B} \sum_{k=0}^{\infty} \frac{\beta^k(Bt - BA)^k}{k!(k + 1)!}$$

$$\times\, e^{-\beta(t-A)}\Theta(t - A). \tag{A1.7}$$

Keeping in mind that hyperbolic Bessel functions of order ν, I_ν, are defined as

$$I_\nu(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{\nu+2k}}{k!\Gamma(\nu + k + 1)} \tag{A1.8}$$

and $\Gamma(k + 1) = k!$, the last sum in (A1.7), after some simple algebraic rearrangements, may be written as

$$(Bt - BA)^{-1/2} I_1[2\beta^{1/2}(Bt - BA)^{1/2}] \qquad t > A.$$

Finally, substituting the last expression into (A1.7) and decoding A, B, α and β we obtain equations (5) and (6) of the main text. Proceeding almost identically, the expression for $n_t(x, s)$ may be found:

$$n_t(x,t) = \frac{n_0}{\mu E\tau(x)}\Theta\left(t - \frac{x}{\mu E}\right)I_0(\xi)\exp\left(-\frac{t - x/\mu E}{\tau_d}\right)\exp\left(-\frac{\bar{S}(x)}{\mu E\tau_0}\right) \tag{A1.9}$$

where ξ, τ_0 and $\bar{S}(x)$ are given by (6)–(8).

In the limiting cases of shallow ($s\tau_d \ll 1$) and deep ($s\tau_d \gg 1$) trapping, (A1.3) may be rewritten as

$$\tilde{n}(x, s) = \frac{n_0}{\mu E}\exp\left[-\frac{s}{\mu E}\left(x + \bar{S}(x)\frac{\tau_d}{\tau_0}\right)\right] \tag{A1.10}$$

and

$$\tilde{n}(x, s) = \frac{n_0}{\mu E}\exp\left(-\frac{\bar{S}(x)}{\mu E\tau_0}\right)\exp\left(\frac{sx}{\mu E}\right) \tag{A1.11}$$

respectively, which immediately yield (10) and (19). Performing integration in (9) with the aid of a theorem on change of variables in δ-functions, one gets (11), (12) and (20).

Appendix 2

Equation (26) may be developed as follows. Taking the Laplace transform of (22) one gets

$$\tilde{n}_t(x, s, \mathscr{E}) = \{C(\mathscr{E})N_0S(x)f(\mathscr{E})/[s + 1/\tau_d(\mathscr{E})]\}\tilde{n}(x, s) \tag{A2.1}$$

which on integration with respect to $\mathscr{E}$ gives

$$\tilde{n}_t(x, s) = \Phi(s)S(x)\tilde{n}(x, s) \tag{A2.2}$$

$$\Phi(s) = \int_0^\infty \frac{C(\mathscr{E})N_0f(\mathscr{E})}{s + 1/\tau_d(\mathscr{E})}\,d\mathscr{E}. \tag{A2.3}$$

As easily seen, (A2.3) is the Laplace transform of (25). Substituting (A2.2) into (A1.1) and solving the resulting equation one gets (26).

Appendix 3

Substitution of (24) into (9) and assuming C and ν to be $\mathscr{E}$-independent one gets

$$j(t) = -\frac{qn_0}{L}\frac{d\Phi(t)}{dt}\int_0^L \exp[-A(x)\Phi(t)]A(x)\,dx \tag{A3.1}$$

where $\Phi(t)$ is given by (25) and $A(x)$ for the STD (15) reads

$$A(x) = (D/\mu E)[1 - \exp(-x/D)]. \tag{A3.2}$$

Introducing the function $F(x, t) = \ln A(x) - A(x)\Phi(t)$, equation (A3.1) may be rewritten as

$$j(t) = -\frac{qn_0}{L}\frac{d\Phi(t)}{dt}\int_0^L \exp[F(x, t)]\,dx. \tag{A3.3}$$

For small times $F(x, t)$ assumes a sharp maximum at the point

$$x^*(t) = -D\ln[1 - \mu E/D\Phi(t)] \tag{A3.4}$$

and may be approximated by

$$F(x,t) \simeq F(x^*(t), t) + \tfrac{1}{2}[x - x^*(t)]^2\,d^2F(x^*(t), t)/dx. \tag{A3.5}$$

Thus evaluation of (A3.3) consists of integration of the Gaussian function (A3.5), which after changing the integration limits from $(0, L)$ to $(-\infty, +\infty)$ leads immediately to equation (34) of the main text. In a similar way equation (35) may be obtained.

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